Q-FiRM: Fidelity-based Rate Maximizing Routes for Quantum Networks

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***Abstract*—Efficient routing of information between end-nodes is a key enabler for secure quantum networks and quantum secret key sharing, which rely on creating and sustaining entangled states over time. However, such pairwise entanglements degrade due to channel loss and the storage of the entangled photons at the network nodes. The state of entanglement in turn impacts *fidelity*, a metric which quantifies the degree of similarity between a pair of quantum states. In this paper, we propose a routing solution that satisfies threshold fidelity requirements imposed by a receiver on the quantum information received from multiple transmitter nodes. Our solution selects intermediate repeaters from a pool of such nodes within the network to maximize the sum-rate of quantum information transfer. To this extent, we first provide expressions for the fidelity loss between adjacent nodes as well as for the end-to-end quantum data rate. Then, we propose a novel two-stage routing solution that (i) identifies the k-shortest paths for each transmitter using fidelity as cost metric and (ii) (heuristically) assigns a path for each transmitter depending on whether the repeater nodes have a single or multiple available memory units. Simulation results demonstrate that our proposed fidelity-based routing solution satisfies a wide range of fidelity requirements [0.6-0.79] while maximizing the quantum information transfer rate, outperforming the existing distance- and hop-based routing approaches.**

***Index Terms*—Quantum networks, quantum repeaters, quantum routing, quantum communication, entanglement**

I. Introduction

The distribution of entangled photons between distant nodes is of vital importance for realizing globally consistent quantum network systems [1], as well as applications, such as quantum secret key sharing [2] and teleportation [3]. This physical process of entanglement couples a pair of particles, such as photons, used to represent quantum bits (qubits) in quantum communication. After entanglement, the state of each qubit cannot be described independently, i.e., an operation on one of them will affect the other, irrespective of the physical distance between them. While the distribution of entangled qubits benefits quantum communications, there exist a number of factors that affect the quality of such entangled states over long distances. This quality is represented by a metric known as *fidelity* that quantifies the degree of similarity between a pair of entangled qubits. Fidelity is impacted by adverse conditions that arise due to transmission channel loss (optical fiber attenuation [4]), quantum memory loss (dephasing [5], and depolarization [6]). It is important to emphasize that these losses exponentially scale with transmission distances and with the storage time of qubits in memory. Thus, to overcome distance-related losses, the use of quantum repeaters that reduce inter-node distances has been widely studied by the research community over the past years [7].

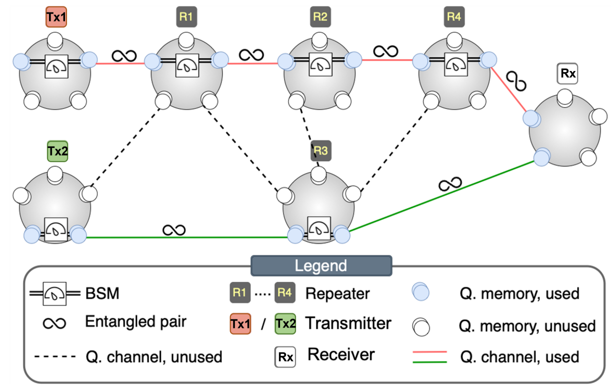


Fig. 1: Network topology with quantum repeater nodes R_1-R_4, transmission channels, and end-to-end paths to receiver Rx shown in bold lines. Paths from transmitters Tx1 and Tx2 have different fidelity requirements F_1 and F_2, respectively.

•**Challenges and Motivations:** The idea of using quantum repeaters raises a number of unique research challenges for the realization of quantum networks. Firstly, the use of a large number of intermediate repeater nodes between transmitter and receiver reduces inter-node distances, which improves fidelity. However, inherent losses at the repeater nodes lower the probability of successfully establishing end-to-end transmission paths, decreasing the quantum data transfer rate. In this context, joint consideration of fidelity and end-to-end data rate while solving a routing problem in memory-equipped quantum repeater networks remains an open challenge. Secondly, in practical quantum networks, nodes may have multiple quantum memory units (QMUs) to store qubits and support multiple simultaneous transmissions. Therefore, to best leverage network resources, routing solutions need to consider the availability of single or multiple QMUs at the repeater nodes. Lastly, there is a need for finding appropriate routing metrics that capture the effects of quantum network imperfections such as memory and fiber losses on the performance of qubit transmission.

These challenges motivate us to answer the following fundamental questions: *(i) How does the repeater selection impact the end-to-end fidelity and data rate of a single path, and (ii) how to design a routing solution that best leverages network* *resources while satisfying fidelity and rate requirements for multiple paths in a quantum network.*

•**Scenario of Interest:** To answer the fundamental questions above, we show in Fig. 1 our scenario of interest. This consists of a quantum repeater network with multiple transmitters and a single receiver. We note, however, that our work can be easily generalized to multi-receiver networks as well. The transmitters and receiver are connected through transmission channels, typically optical fiber for distributing the entangled pairs. Each repeater node is equipped with QMUs, which are used to perform Bell State Measurements (BSM) for interconnecting pairs of repeater nodes. Moreover, each transmitter may serve a different application. Depending on the application, qubits need to meet different fidelity requirements at the receiver end.

•**Proposed Q-FiRM Solution:** To address these unique challenges for the scenario of interest, we propose Q-FiRM, for Fidelity-based Rate Maximizing routes for Quantum networks. As a route selection algorithm, Q-FiRM maximizes the sum-rate of qubit transfers for all transmitter-receiver paths, where the received qubit from each transmitter is subject to a different fidelity requirements set by the receiver. Specifically, we propose a two-stage solution that first finds feasible paths for each transmitter. Then, it selects one of the feasible paths for each transmitter-receiver transmission flow. This selection considers the availability of single or multiple QMUs at the repeater nodes. For finding the paths, we modify the existing k-shortest path routing algorithm, by using fidelity as a routing metric. To this extent, we first provide the expressions for the end-to-end fidelity and sum-rate that we then leverage in our proposed Q-FiRM routing solution.

•**Summary of Contributions:**

1) We propose a two-stage routing solution Q-FiRM that assigns paths to multiple transmitters in a quantum repeater network by maximizing the sum-rate of quantum information transfer subject to meeting the threshold fidelity requirements that are imposed by the receiver on per transmitter basis.
2) We consider in the routing solution single or multiple quantum memory units at each of the repeater nodes, which in turn improves usage of network resources.
3) We propose the use of fidelity as the cost metric in the classical k-shortest path routing algorithm. This metric accounts for memory and fiber losses that affects qubits along the paths in the network.
4) Simulation results demonstrate that our proposed fidelity metric-based routing solution for rate maximization provides an improvement of upto 24.8% and 12% in sum-rate compared to the traditional hop- and distance-based routing solutions, respectively.

We organize the remainder of the paper as follows: we review existing works related to repeater networks in Section II. Then, in Section III, we discuss fidelity and transmission rate metrics. In Section IV we present our proposed Q-FiRM routing solution, followed by simulation results in Section V. Lastly, we conclude the paper in Section VI.

II. Related Work

This section discusses the fundamental concepts of repeater and related works on routing solution in a quantum network.

• **Quantum Repeater:** A quantum repeater uses an entanglement swapping protocol that generates new long distance entanglement using two shorter distance entanglements. In this operation, the repeater node conducts the BSM on two qubits, one from each entangled pair. The resulting classical information output is used for the correction of the remaining two qubits of each pair so that they become entangled [8]. This entanglement swapping mitigates the transmission channel loss inherent in achieving direct long distance entanglement [9].

• **Quantum Repeater Network:** A quantum repeater network consists of multiple repeater nodes connected via optical fiber transmission channels. Each repeater node is capable of storing incoming qubits and conducting the BSM operations on them. Several quantum repeater protocols have been proposed for optimizing rate or fidelity. The works in [10], [11] focus on estimating transmission waiting time and fidelity without considering cumulative losses inherent in transmission channels and quantum memories. Similarly, the authors in [12] propose a rate-centric shortest path entanglement routing protocol for a square grid network topology. Further, the work in [13] develops a routing algorithm that exhaustively finds all paths between transmitter and receiver and then selects the path using a custom end-to-end rate metric. The above works do not consider the effect of network imperfections on fidelity of the entangled qubits, which plays an important role in routing.

III. Performance Metrics

In this section, we outline end-to-end fidelity and rate metrics for a given path in a quantum repeater network, which we use in our proposed Q-FiRM routing solution (Sec. IV).

A. Fidelity Metric

Fidelity is a metric that quantifies similarity between states of two qubits. For the case of pure quantum state [14] considered in this work, the fidelity is calculated as $F = \langle\Psi|\rho|\Psi\rangle \in [0, 1]$. Here, $|\Psi\rangle$ is a reference qubit state, and ρ is density matrix of one of the qubit state of an entangled pair received at a node, which is impacted by the adverse transmission environment. Statistically, the degradation of ρ is modeled using stochastic Pauli Operators [15] with different weights. The attenuation effect of transmission channel (optical fiber) on this received qubit's density matrix is modeled as [4]:

$$\begin{aligned}\rho_{fiber}(\rho) = (1 - 0.75p_{fiber})I\rho I^\dagger + 0.25p_{fiber}X\rho X^\dagger \\ + 0.25p_{fiber}Y\rho Y^\dagger + 0.25p_{fiber}Z\rho Z^\dagger,\end{aligned} \tag{1}$$

where I, X, Y, and Z are Pauli Operators [15]. p_{fiber} denotes the probability of loss of this qubit in the fiber, which is given by [4]:

$$p_{fiber} = 1 - (1 - p_{init}) \cdot 10^{(-\eta l/10)}, \tag{2}$$

where l is the distance over which the qubit is transmitted, p_{init} is the probability that the qubit is lost after its generation, and η is fiber attenuation factor.

The quantum memory results in dephasing and depolarization of this received qubit. The corresponding effects on its density matrix are modeled as [4]:

$$\begin{aligned}\rho_{dephase}(\rho) = (1 - p_{dephase}) I\rho I^\dagger + 0 p_{dephase} X\rho X^\dagger \\ + 0 p_{dephase} Y\rho Y^\dagger + p_{dephase} Z\rho Z^\dagger,\end{aligned} \tag{3}$$

$$\begin{aligned}\rho_{depol}(\rho) = (1 - 0.75 p_{depol}) I\rho I^\dagger + 0.25 p_{depol} X\rho X^\dagger \\ + 0.25 p_{depol} Y\rho Y^\dagger + 0.25 p_{depol} Z\rho Z^\dagger.\end{aligned} \tag{4}$$

The parameters $p_{dephase}$ and p_{depol} denote the probabilities of dephasing and depolarization, respectively, and are given by

$$p_{dephase} = 1 - e^{-\triangle t \cdot R_{dephase}}, \tag{5}$$

$$p_{depol} = 1 - e^{-\triangle t \cdot R_{depol}}, \tag{6}$$

where $R_{dephase}$ represents the dephasing rate (in Hz), R_{depol} denotes the depolaring rate (in Hz), and Δt is the time qubit is in the memory.

The cumulative effect of these losses on the density matrix and fidelity of the qubit transferred over a transmission channel n are given by:

$$\rho_n = \rho_{depol}(\rho_{dephase}(\rho_{fiber}(\rho))), \tag{7}$$

$$F_n = \langle \Psi | \rho_n | \Psi \rangle . \tag{8}$$

For a transmission channel obtained by connecting two adjacent transmission channels (with entangled pairs' fidelities F_n and F_{n+1}) using a repeater, the resulting fidelity $F_{n,n+1}$ is given by [1]:

$$F_{n,n+1} = F_n F_{n+1} + \frac{(1 - F_n)(1 - F_{n+1})}{3}. \tag{9}$$

This expression is recursively applied to calculate end-to-end fidelity $F_{r(m)}$ of the qubit transmitted over a path $r(m)$ from the transmitter m to the receiver, with multiple transmission channels in between due to the use of repeaters.

B. End-to-End Rate Metric

The end-to-end quantum data rate $R_{r(m)}$ between the transmitter m and receiver connected through a path $r(m)$ with N repeaters (i.e., $(N + 1)$ transmission channels) is calculated by considering the total time $T_{r(m)}$ required for achieving successful teleportation (via end-to-end entanglement swapping) of the transmitter's qubit to the receiver. It comprises of delays encountered in classical and quantum information transfers (τ_c, τ_q), the BSM operations (τ_{BSM}), and the qubits corrections for the reconstruction of the information at the receiver (τ_d) [3], given in Eq. 10. For a path with total distance L, the total delay due to classical information transfer is $\tau_c = \frac{L}{c}$ seconds and that due to entangled pair distribution is $\tau_q = \frac{l}{c_{fiber}}$ seconds, where l is the length of the longest transmission channel for qubit transfer among all the transmission channels present in the considered path. Thus, the total time required for successful transmission of qubit over a path is given by

$$T_{r(m)} = \tau_c + \tau_q + \tau_{BSM} + \tau_d. \tag{10}$$

In a practical environment, the noisy quantum memory and optical fiber impact the end-to-end rate. Thus, probability of successful transfer of a qubit over a quantum channel n of length l is given by $p_n^{succ} = (1 - p_{fiber})(1 - p_{dephase})(1 - p_{depol})$. For a path $r(m)$ from transmitter m to a receiver comprising $(N+1)$ quantum channels, this success probability is written as $P_{r(m)}^{succ} = \prod_{n=1}^{N+1} p_n^{succ}$ in case of perfect BSM operations. While for imperfect BSM operations, $P_{r(m)}^{succ} = \left(\prod_{n=1}^{N+1} p_n^{succ}\right) (p_{BSM}^{succ})^N$, where p_{BSM}^{succ} represents a finite probability of successful BSM operation at a node and its exponent N denotes the number of BSM operations conducted in the given path $r(m)$. Note that for the perfect (imperfect) BSM, p_{BSM}^{succ}=1 (< 1). The resulting end-to-end rate is given by,

$$R_{r(m)} = \frac{P_{r(m)}^{succ}}{T_{r(m)}}. \tag{11}$$

The sum-rate of quantum data transfer from all the M transmitters to a receiver is calculated as:

$$R_{total} = \sum_{m=1}^{M} R_{r(m)}. \tag{12}$$

A summary of all the variables used in rate and fidelity calculations is provided in Table I along with their values used for generating simulation results in Sec. V (similar to work [4]).

TABLE I: Symbols used in Rate and Fidelity Expressions

Symbol	Description	Value
c	Speed of classical information transfer over air	3e8 km/s
c_{fiber}	Speed of quantum information transfer over fiber	2e8 km/s
τ_{BSM}	Time delay in memory for BSM	10e-9 s
τ_d	Time delay for qubit correction	10e-9 s
η	Fiber attenuation factor	0.15 dB/km
P_{init}	Probability of loss of entangled pair after generation	0.1
$\triangle t$	Time for which a qubit is stored in memory	10e-9 s
$R_{dephase}$	Depahsing rate of a qubit in memory	10e6 Hz
R_{depol}	Depolaring Rate rate of a qubit in memory	10e3 Hz

IV. Proposed Quantum Routing

In this section, we first formulate the routing problem for a quantum network. Then, we introduce Q-FiRM, our proposed solution to ensure successful quantum data transmission from multiple transmitters to a single receiver.

A. Routing Problem Formulation

We consider a network composed by M transmitters, a single receiver, and a number of inter-connected quantum repeater nodes, as we show in Fig. 1. The receiver imposes a different fidelity requirement on the qubits received from each transmitter, which is given by,

$$F_{r(m)} \geq F_{c(m)}, \tag{13}$$

where $F_{r(m)}$ is the final fidelity calculated from Eq. 9 for transmitter m and a given end-to-end path $r\,(m)$, and $F_{c(m)}$ is the minimum required fidelity for the qubit received from the transmitter m, determined by the application at the receiver end. We compute the M routing paths for all transmitter-receiver pairs that maximize the sum-rate of quantum data transfer (given by Eq. 12) subject to the individual fidelity constraints in Eq. 13. Thus, we aim to solve the following optimization problem:

$$\underset{r(1),\cdots,r(M)}{\text{maximize}} \quad \sum_{m=1}^{M} R_{r(m)} \tag{14a}$$

$$\text{subject to} \quad F_{r(m)} \geq F_{c(m)}, \quad m = 1, 2, \cdots, M. \tag{14b}$$

B. Proposed Routing Solution

Next, we introduce our solution to the problem presented in Eq. 14. We propose a modified version of the k-shortest path routing algorithm, which is an extension of the traditional Dijkstra routing algorithm [16]. Specifically, our solution identifies the k routing paths with maximum fidelity in a quantum network. This solution differs from the classical k-shortest path routing algorithm that finds the k paths of minimum length. However, considering fidelity as metric in the k-shortest path algorithm alone does not guarantee that (i) minimum fidelity requirements are satisfied for each of the M transmitters, as given by Eq. 13, and (ii) the end-to-end quantum data rate is maximized.

One solution to address (i) and (ii) is to enumerate all paths from each of the M transmitters to the receiver [17], identify *feasible* paths by eliminating those not satisfying fidelity constraints, and select the path from the remaining set of feasible paths that provides the highest rate. An alternative solution is to set k to an arbitrary high value and then check fidelity constraints. However, none of these solutions are scalable with the network size. Thus, to provide a scalable routing solution to address (i) and (ii), we propose Q-FiRM that we describe next as the following two-stage approach:

1) A Scalable Constrained Fidelity-based Routing Solution: In the first stage of Q-FiRM, we run our modified fidelity metric-based k-shortest paths algorithm for each of the M transmitters. For each transmitter, we start from a value of $k = 1$. After each run, we check the condition given in Eq. 13 for all k paths. If all paths satisfy Eq. 13, we increase the value of k in one unit and re-run the k-shortest paths algorithm until at least one path does not satisfy Eq. 13. In such case, all feasible paths for transmitter m are found. The triggering of this stopping condition for all M transmitters finalises stage one, providing a list of feasible paths from each of the M transmitters to the single receiver. We note that this approach alleviates the scalability issue that persists on exhaustive search-based algorithms [17].

2) Sum Data Rate Maximization: The goal in stage two is to assign one of the paths found in stage one to each of the M transmitters, such that the sum data rate is maximized. We have considered two network settings, in which nodes are either equipped with multiple or a single QMU. In the former case, paths from multiple transmitters to the receiver can overlap, i.e., paths assigned to different transmitters can share repeater nodes. In this case, the selected path for transmitter m is the path that maximizes the sum-rate among all feasible paths for this transmitter found in stage one. Thus, the path selection is done on a per-transmitter basis, with no dependence on other transmitters selection. We note that in the multiple QMUs case, we maximize the sum-rate given by Eq. 12 and find the global maximum to the problem in Eq. 14.

Algorithm 1 Proposed Routing Solution: Q-FiRM

Input: $M, F_{c(m)}, k_0(m) = 1, m = 1, \cdots, M$.
Output: $r(m), \forall m$: Assigned path for M transmitters.
1. **Stage** 1:
2. $r_c(m) = [\ ], \forall m$. (Stores feasible paths)
3. **for** $m = 1, \cdots, M$
4. $r_{tmp} = [\ ], F_{tmp} = 1$
5. **while** $F_{tmp} >= F_{c(m)}$
6. Obtain route r_{tmp} from fidelity-metric based k-shortest path algorithm with $k = k_0(m)$.
7. Calculate Fidelity F_{tmp} for route r_{tmp} using Eq. 9.
8. **if** $F_{tmp} >= F_{c(m)}$
9. $k_0(m) = k_0(m) + 1,\ r_c(m) = [r_c(m)\ r_{tmp}]$.
10. **end**
11. **end**
12. **end**
13. **Stage** 2:
14. Calculate end-to-end rate $R_r, \forall r \in r_{all} = \{r_c(m), \forall m\}$ (Eq.11).
15. **Nodes with multiple memory units**
16. For each transmitter m, select path $r(m)$ with the highest $R_{(\cdot)}$ from its feasible paths list $r_c(m)$.
17. **Nodes with single memory unit (Greedy approach)**
18. Select one path with the highest rate from list of all feasible paths r_{all}.
19. Assign this path to $r(\cdot)$ of its starting node (i.e., transmitter).
20. Update list r_{all} by removing paths (i) overlapping with the selected path and (ii) starting from transmitter of the selected path.
21. Repeat this path selection for other transmitters until either all M transmitters are assigned paths or the list r_{all} becomes empty.

In the single memory case, we provide a heuristic solution to the problem in Eq. 14. We consider a Greedy approach to select disjoint paths from each transmitter to the receiver that maximize the sum-rate. To do so, we first choose the path that gives maximum rate from the complete list of all feasible paths for all M transmitters. Then, we update this list by removing i) overlapping paths with the chosen one that share at least one node and ii) paths departing from the transmitter node of the chosen path. We iteratively perform path selection and feasible path list update until either we assign a path to all M transmitters or the list of feasible paths is empty.

We summarize our proposed two-stage Q-FiRM routing solution in Algorithm 1.

C. Complexity Analysis

In this section, we evaluate the worst-case complexity of our proposed two-stage routing solution.

Stage One: The complexity in stage one is dictated by (i) the k shortest path algorithm and (ii) the check of fidelity constraint to identify feasible paths. The complexity of Yen's algorithm for solving the k-shortest path problem is given by $\mathcal{O}(kv(a + v\log(v)))$ [18], with v the number of vertices (repeater nodes) and a the number of arcs (link connections). The worst-case complexity in stage one corresponds to the case where all paths found for any value of k satisfy fidelity requirements. Then, new searches are sequentially triggered for a one-unit increased value of k, until the fidelity requirement is violated at value k_{max}. Thus, the complexity of the k shortest path algorithm for stage one is given by $M\mathcal{O}(k_{max}v(a+v\log(v)))$. Moreover, the complexity of checking fidelity constraints is $M\mathcal{O}(k_{max})$. Then, the total worst-case complexity for stage one is $M(\mathcal{O}(k_{max}v(a+v\log(v)))+\mathcal{O}(k_{max}))$.

Stage Two: For the case where nodes have multiple QMUs, we perform M sorting operations of size k_{max} using *MergeSort* obtaining a worst-case complexity given by $M\mathcal{O}(k_{max}\log(k_{max}))$. For the case where nodes have a single QMU, the worst-case occurs when the remaining feasible paths after each path selection do not share common nodes with the already chosen paths. Therefore, at each iteration of the Greedy approach, sorting operations and updates of the list of feasible paths need to be performed. This provides an upper bounded worst-case complexity of $M\mathcal{O}\left(Mk_{max}log\left(Mk_{max}\right)\right)+M\mathcal{O}\left(Mk_{max}\right)$.

The overall worst-case complexity of Q-FiRM for multiple QMUs is $M\mathcal{O}(k_{max}v(a+v\log(v)))+M\mathcal{O}(k_{max}\log(k_{max}))$ and for single QMU is $M\mathcal{O}(k_{max}v(a+v\log(v)))+M\mathcal{O}\left(Mk_{max}log\left(Mk_{max}\right)\right)$.

For practical quantum networks with multiple QMUs at each repeater node, the worst-case complexity of an algorithm in [13] that first enumerates all paths connecting each transmitter to the receiver (k_{all}) and then selects the paths that maximize sum-rate is $M\mathcal{O}(k_{all}v(a+v\log(v)))+M\mathcal{O}(k_{all}\log(k_{all}))$. Our proposed Q-FiRM routing solution helps solve the scalability problem of such exhaustive-search based approaches when $k_{max} << k_{all}$, which results from the imposed fidelity constraints in Eq. 14b.

V. Simulation Results

In this section, we compare the performance of Q-FiRM with distance- and hop-based routing solutions in a quantum repeater network. We consider scenarios in which repeater nodes are equipped with single and multiple QMUs.

A. Description of Quantum Repeater Network

We consider a quantum network topology that consists of 15 nodes placed randomly in an area of 20×20 km^2, as shown in Fig. 2. In this network, a direct link between two nodes is established if the inter-node distance is less than 10 km. Moreover, we consider up to 5 transmitter nodes and one receiver node with indexes $\{2,4,6,7,5\}$ and $\{15\}$, respectively. We choose the receiver node such that it has sufficient number of incoming links, which enables it to receive data from multiple transmitters. Further, we consider two scenarios where all the nodes are equipped with either single or multiple QMUs. We note that Q-FiRM can find routes for any generic network topology. Further, it can find paths for any number of transmitters and receivers while offering maximum sum-rate and satisfying fidelity requirements for received qubits. We use Matlab for simulating the quantum repeater network and

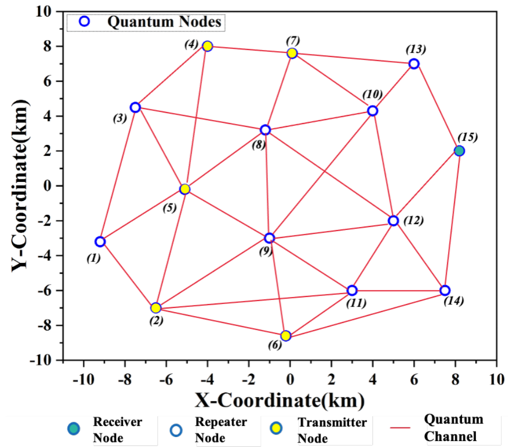


Fig. 2: Quantum Network

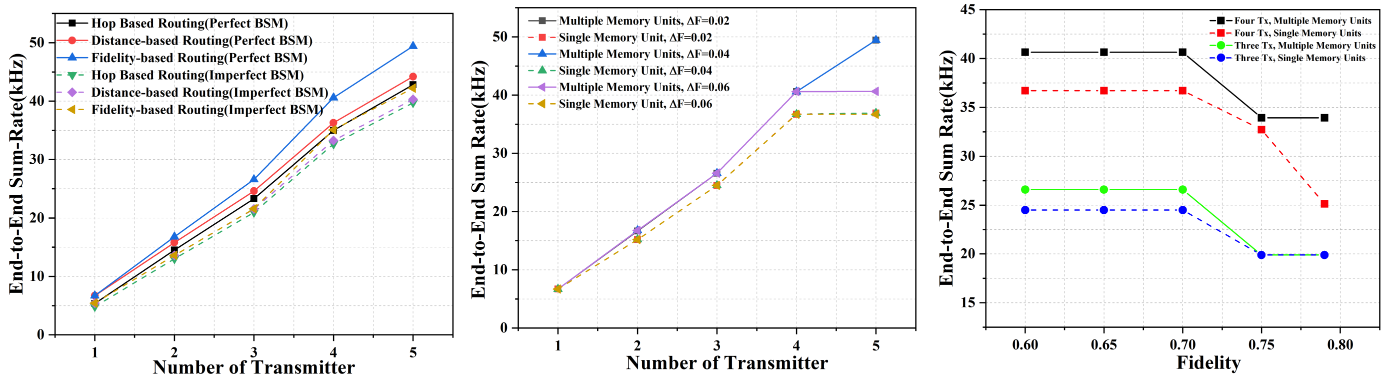
Q-FiRM routing solution. We write functions for calculating fidelity and rate metrics that are called by Q-FiRM for paths selection. Further, the considered values of network parameters are shown in Table I.

B. Comparison of Q-FiRM with Existing Routing Schemes

In Fig. 3a, we compare sum-rates of quantum information transfer obtained using our proposed fidelity metric-based rate maximization routing with that obtained using the existing hop- and distance-based routing metrics under the scenarios of perfect and imperfect BSMs. The hop-based approach provides path with minimum number of repeater nodes between the transmitter and receiver, while the distance-based gives the shortest distance path between them. We observe lower sum-rates in case of the imperfect BSMs (dashed line) compared to the perfect BSMs case (solid line), but these two scenarios have same trend of the sum-rate change for the different routing metrics. Thus, in the following results, for the purpose of simplicity, perfect BSMs are considered in the results of Fig. 3b and Fig. 3c. Further, we observe that our fidelity cost metric-based Q-FiRM provides paths with upto 24.8% and 12% better sum-rates compared to the hop- and distance-based approaches, respectively. For fair comparison, we set the fidelity requirements for qubits received from different transmitters in our approach similar to what is obtained in the comparative approaches. We provide this comparison for multiple QMUs, which represents a practical quantum network with each repeater node capable of routing multiple qubits.

C. Routing Performance: Single vs. Multiple QMUs at Each Repeater Nodes

We want to highlight that our proposed Q-FiRM selects disjoint (overlapping) paths that maximizes sum-rate of qubits transmission from different transmitters to a single receiver in case of single (multiple) QMU(s) at the repeater nodes. In the former case, the disjoint path selected for each transmitter to achieve maximum sum-rate may not be the best path rate-wise for that transmitter. This results in lower achievable sum-rates with a single QMU at each repeater node compared to multiple QMUs case, as shown with dotted and solid lines in Figs. 3b and 3c.



(a) (b) (c)

Fig. 3: (a) Sum-rate comparison of Q-FiRM with existing schemes ($p_{BSM}^{succ} = 0.9$ for imperfect BSM operations case). Sum-rate versus (b) number of transmitters and (c) fidelity plots for a quantum network with single and multiple QMUs at repeater nodes considering perfect BSM operations ($p_{BSM}^{succ} = 1$).

In the Fig. 3b, we observe that on increasing the number of transmitters, the amount of increase in sum-rate is more for multiple QMUs case. Also, due to increased number of transmissions in the network, the increase in sum-rate is intuitive. Further, in this figure, different fidelity requirements for qubits received from each transmitter are set in step increments of $\Delta F = \{0.02, 0.04, \text{ and } 0.06\}$, starting from fidelity 0.6 for qubit received from first transmitter. We observe that imposing strict fidelity requirements with increased number of transmissions limits sum-rate even in case of multiple QMUs.

In Fig. 3c, we consider a scenario where the receiver requires the same fidelity for qubits received from each transmitter. We observe that a lesser sum-rate is achieved in satisfying high fidelity requirements of $F = 0.75$ for each of the 4 transmitted qubits compared to the unequal fidelity requirements of $F = \{0.6, 0.66, 0.72, 0.78\}$ (case $\Delta F = 0.06$) for the 4 transmitters case in Fig. 3b. These results also validate well-known rate-fidelity trade-off present in the quantum networks. Additionally in Figs. 3b and 3c, we observe constant rate with different fidelity since the same paths are chosen under the constrained optimization.

VI. Conclusion

In this paper, we propose Q-FiRM, a fidelity metric-based routing solution for quantum repeater networks that considers the availability of single or multiple QMUs at the repeater nodes. Our solution finds the paths from multiple transmitters to a single receiver that maximize the sum-rate of quantum information transfer while satisfying different fidelity requirements for qubits received from each transmitter. To this extent, we provide end-to-end fidelity and rate expressions for each path that are leveraged in our proposed Q-FiRM solution as QoS metrics. We demonstrate that Q-FiRM provides upto 24.8% and 12% improvement in sum-rate compared to traditional hop- and distance-based routing schemes, respectively.

References

[1] W. Kozlowski, A. Dahlberg, and S. Wehner, "Designing a quantum network protocol," in *Proceedings of the 16th International Conference on emerging Networking EXperiments and Technologies*, 2020, pp. 1–16.

[2] M. Hillery, V. Bužek, and A. Berthiaume, "Quantum secret sharing," *Physical Review A*, vol. 59, no. 3, p. 1829, 1999.

[3] D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, "Experimental quantum teleportation," *Nature*, vol. 390, no. 6660, pp. 575–579, 1997.

[4] C. Cicconetti, M. Conti, and A. Passarella, "Request scheduling in quantum networks," *IEEE Transactions on Quantum Engineering*, vol. 2, pp. 2–17, 2021.

[5] G. A. Paz-Silva, S.-W. Lee, T. J. Green, and L. Viola, "Dynamical decoupling sequences for multi-qubit dephasing suppression and long-time quantum memory," *New Journal of Physics*, vol. 18, no. 7, p. 073020, 2016.

[6] C. Cafaro and S. Mancini, "Quantum stabilizer codes for correlated and asymmetric depolarizing errors," *Physical Review A*, vol. 82, no. 1, p. 012306, 2010.

[7] Z.-D. Li, R. Zhang, X.-F. Yin, L.-Z. Liu, Y. Hu, Y.-Q. Fang, Y.-Y. Fei, X. Jiang, J. Zhang, L. Li *et al.*, "Experimental quantum repeater without quantum memory," *Nature photonics*, vol. 13, no. 9, pp. 644–648, 2019.

[8] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, "Quantum entanglement," *Reviews of modern physics*, vol. 81, no. 2, p. 865, 2009.

[9] Z. Zhao, T. Yang, Y.-A. Chen, A.-N. Zhang, and J.-W. Pan, "Experimental realization of entanglement concentration and a quantum repeater," *Physical review letters*, vol. 90, no. 20, p. 207901, 2003.

[10] S. Guha, H. Krovi, C. A. Fuchs, Z. Dutton, J. A. Slater, C. Simon, and W. Tittel, "Rate-loss analysis of an efficient quantum repeater architecture," *Physical Review A*, vol. 92, no. 2, p. 022357, 2015.

[11] S. Abruzzo, S. Bratzik, N. K. Bernardes, H. Kampermann, P. Van Loock, and D. Bruß, "Quantum repeaters and quantum key distribution: Analysis of secret-key rates," *Physical Review A*, vol. 87, no. 5, p. 052315, 2013.

[12] M. Pant, H. Krovi, D. Towsley, L. Tassiulas, L. Jiang, P. Basu, D. Englund, and S. Guha, "Routing entanglement in the quantum internet," *npj Quantum Information*, vol. 5, no. 1, pp. 1–9, 2019.

[13] M. Caleffi, "Optimal routing for quantum networks," *IEEE Access*, vol. 5, pp. 22 299–22 312, 2017.

[14] Y.-C. Liang, Y.-H. Yeh, P. E. Mendonça, R. Y. Teh, M. D. Reid, and P. D. Drummond, "Quantum fidelity measures for mixed states," *Reports on Progress in Physics*, vol. 82, no. 7, p. 076001, 2019.

[15] Y.-S. Kim, H.-T. Lim, Y.-S. Ra, and Y.-H. Kim, "Experimental verification of the commutation relation for pauli spin operators using single-photon quantum interference," *Physics Letters A*, vol. 374, no. 43, pp. 4393–4396, 2010.

[16] J. Y. Yen, "Finding the k shortest loopless paths in a network," *Management Science*, vol. 17, no. 11, pp. 712–716, 1971.

[17] F. Rubin, "Enumerating all simple paths in a graph," *IEEE Transactions on Circuits and Systems*, vol. 25, no. 8, pp. 641–642, 1978.

[18] J. Hershberger, M. Maxel, and S. Suri, "Finding the k shortest simple paths: A new algorithm and its implementation," *ACM Transactions on Algorithms (TALG)*, vol. 3, no. 4, pp. 45–es, 2007.