BiP: Bit-Phase-Flip Error Mitigation in Quantum Communications

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***Abstract*—Quantum links are inherently noisy and quantum information bits (qubits) suffer upto 13% degradation in their entangled states within time-scale of 0.5 ms. Thus, mitigating errors becomes essential for reliable end-to-end data communication in a multi-hop quantum network. Compared to the typical operations performed within the contained environment of a single quantum computer, removal of both bit- and phase-flip errors in a distributed network of such computers is challenging due to the stochastic variations in the noise at each intermediate link. This paper describes a scheme that determines both the bit- and phase-flip errors (abbreviated as 'BiP') and mitigates them for distributed and networked quantum systems. To achieve this, we model the environment noise using general error models and obtain error calibration matrices in different computational bases for bit-phase-flip errors. Results reveal that BiP improves the fidelity beyond 95% for the received qubits compared to the state-of-the-art error mitigation method by correcting the elevation θ and azimuthal angles φ in the Bloch sphere representation.**

***Index Terms*—Quantum communication network, quantum error mitigation, quantum computing**

I. Introduction

A quantum communication network is a key enabler for distributed quantum computing [1] and quantum cryptography [2]. For multi-hop network composed of entangled quantum information bits (qubits), we need to overcome the adverse effects of decoherence in the state of these qubits. Typically, the probability of decoherence in phase can reach upto 33% for qubits entangled using process spontaneous parametric down-conversion (SPDC) in nonlinear $\chi^{(2)}$ crystals , which introduces bit flips at rate of 1 to 400 Hz [3], ultimately degrading the link quality. Thus, creating a reliable quantum error estimation and correction approach that can mitigate the decoherence between entangled qubits is important and the first step towards building a fault-tolerant, large-scale distributed quantum information processing system.

•**Current state-of-the-art:** Noisy-intermediate scale quantum (NISQ) computers today already incorporate fault-tolerant operations in the form of error correction algorithms [4]. This specific approach increases resiliency of quantum information transfer by encoding multiple physical qubits into one logical qubit. However, this algorithm comes with an enormous hardware cost owing to the requirement of extra entangled pairs for encoding logical qubits apart from teleportation in large scale quantum communication system. Error-correctable quantum teleportation [5] has experimentally shown the ability to teleport one logical qubit, but with the mandatory requirement of a polarization-entangled four-photon GHZ (GHZ_4) state

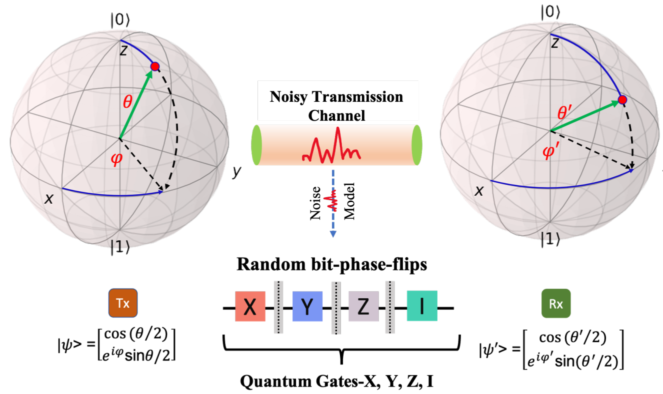


Fig. 1: Rotation in elevation θ and azimuthal φ angles of the qubit state in the Bloch sphere during the noisy transmission from Tx to Rx. The noise arises due to qubit decoherence in fiber-based transmission link and is represented by an error model implemented using quantum gates $X, Y, Z,$ and I.

generation and the corresponding multiple Bell State Measurements (BSMs). These operations have high dependencies on complicated hardware, such as extra three nonlinear crystals and eight single photon detectors (SPDs), and require complex analysis during three consecutive stages of error measurement and 24 SPDs for the logical qubit readout stage. Thus, an approach with less dependency on additional hardware-related analysis for error correction in logical qubits is desired. Ideally, an error mitigation strategy must compensate for errors during transmission, while being independent of the physical device requirements and the encoding method used to map the logical qubits [6]. The standard approach followed in quantum computing paradigms, such as use-cases in quantum chemistry [7], and dynamic quantum simulations [8], is to initialize a calibration measurement with a given error model to get the calibration matrix, which is then used to eliminate the effect of noise on the qubits measurement during computing. This cannot directly correct the rotation in angles of the distorted qubits, which is required for correctly decoding the information encoded in these phase angles of the qubits in the quantum communication use-cases.

•**Motivation and novelty of our approach:** Error mitigation method in the above-mentioned applications can synchronously address only bit-flip errors in multiple qubits. However, such bit-flip error mitigation approaches are presently limited only to the z computational basis, which ignores the phase change of the states within qubits, where both bit and phase are the essential metrics for the correct characterization of the received qubits in quantum communication. Therefore, an error mitigation scheme that includes calibration in x, y, and z computational bases is required to jointly handle the bit- and phase-flip errors. This will accurately capture the changes in the states of the qubit in a noisy environment, which then improves the fidelity of the link. Additionally, an error mitigation approach becomes richer when models of depolarization and dephasing are added to the baseline, which include decoherence due to the noise incurred in the fiber channel and gate operations, respectively. These two parameters are mathematically modeled using Pauli operators (X, Y, Z, I) with different probabilities. The overall effect of this type of decoherence results in change of the elevation θ and azimuthal φ angles of the received qubits along z and x directions in Bloch sphere as shown in Fig. 1. In summary, our proposed error mitigation approach- BiP is carefully designed to include the enhancements of (i) using additional (x, y) computational bases, (ii) modeling the effect of noise inherent in quantum communication network in the calibration and measurement processes, as opposed to the state-of-the-art that considers the noise in the quantum computing applications [9], and (iii) correcting the angles of the distorted qubits which may contain phase-encoded information.

•**Proposed BiP Solution:** BiP eliminates the random bit-phase-flip errors in qubits transmitted within a noisy communication channel. BiP takes x, y, and z bases measurements, and the usage of ancillary qubits, which enables comprehensive mitigation of the changes in the angles of the qubit state in the Bloch sphere. The ancillary qubits are extra qubits used to infer the information of the received qubit without direct measurements. We compare BiP with the state-of-the-art bit- and phase-flip approaches and show that our solution provides better fidelity for the received qubits.

•**Summary of Contributions:** The main contributions of the BiP error mitigation approach are as follows:

1) We propose a bit-phase-flip error mitigation approach called BiP for mitigating errors in qubits during transmission through noisy quantum communication channels. BiP can be readily applied to any existing quantum communication system without requiring any additional hardware such as SPDs.
2) We design a receiver-side quantum circuit for calibration measurement, which generates different calibration matrices in different computational bases for accurate calculation of the changes in two angles of the received qubits. This approach advances the analytical expressions for mitigation of changes in qubits' angles, which is typically done with only one computational basis in the state-of-the-art.
3) We design a receiver-side quantum circuit that infers information of the received qubits (without measuring them) using ancillary qubits and performs measurements on these qubits in different computational bases.
4) We provide theoretical analysis of the calibration and ancillary qubits measurements used in BiP for mitigating the changes in the two angles and improving the fidelity of the received qubit. Simulation results show that BiP achieves fidelity of the received qubit ≥ 0.95 for different fiber transmission-channel lengths.

II. Related Work

In this section, we survey the state-of-the-art methods for addressing the impact of noise at the qubits during transmission from perspective of the physical and link layers [10].

For long-distance and large-scale quantum communication, the primary source of decoherence of qubits is attenuation by the optical fiber, which increases exponentially with the length of the fiber. To overcome this loss, repeaters-based methods divide a long-distance optical fiber into small segments connected by quantum repeater nodes However, these repeaters increase the complexity of overall operation and also introduce new errors while processing qubits.

Quantum error correction (QEC) is another emerging field to protect qubits from noisy environment in long-distance quantum communication. It leverages repetition coding algorithm, which encodes multiple physical qubits in one logical qubit for error detection and correction. This method protects logical qubit information by reducing the weighted error probability at the cost of multiple qubits, such as GHZ_4 for teleportation of one logical qubit [5]. However, generating entanglement with multiple qubits involves complicated physical device operation and also the complex measurements on multiple qubits increase the burden of error correction.

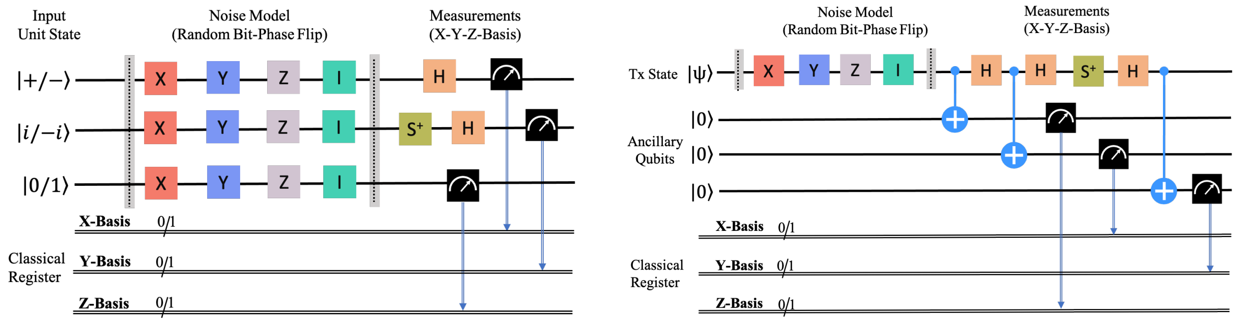
Quantum error mitigation is increasingly getting more attention for the error processing in the area of quantum computing. It compensates for decoherence in qubits by applying a calibration matrix with random error model. The authors in [9] propose bit-flip-average method for mitigation of bit-flip errors in qubits. However, conventional error mitigation methods are unable to completely describe the qubit information for long distance quantum communication, since the phase information of qubits are not considered.

III. Theoretical Analysis for Error Mitigation Model with bit-phase-flip

In this section, we outline the theory associated with our proposed BiP error mitigation method. First, we introduce the error model used for characterizing environmental noise experienced by qubits during transmission via quantum communication system. Next, considering this error model, we describe our design of quantum circuits to obtain the calibration matrix and the measurement of received qubit in different computational basis, both of them are conducted on QISKIT [11]. Then, we explain our mitigation process that corrects the elevation and azimuth angles of the noisy qubits.

A. Error Model

In a quantum communication system, qubits suffer degradation while propagating through quantum channels, which are generally noisy in nature. This noise-induced degradation results from the attenuation of qubits in the optical fiber-based transmission channel and their depolarization and dephasing



(a) (b)

Fig. 2: (a) Calibration measurement circuit in different computational bases for computing calibration matrices $\mathbf{M}_{(\cdot)}$. (b) Circuit for measuring the received noisy qubit using ancillary qubits.

inside the quantum nodes. Statistically, this degradation of a qubit is modeled using an error operator $\varepsilon(p)$, which is a function of weighted Pauli operators. The error operator operating on a random qubit state $|\psi\rangle$ is expressed as [12]–[15]:

$$\varepsilon(p) = \sum_{i=1} p_i O_i \rho O_i^\dagger, \tag{1}$$

where O_i represents the i^{th} operator- X, Y, Z, or I Pauli operators, ρ is the density matrix of the given qubit state $|\psi\rangle$, calculated as $\rho = |\psi\rangle\langle\psi|$, and p_i denotes the weight of each operator. These weights are calculated as the probability of degradation of qubit due to the 1) attenuation in the fiber (p_{fiber}) and 2) dephasing and depolarizing originated from the gate operations in the quantum nodes ($p_{dephase}$ and p_{depol}). Specifically, the error operator due to fiber attenuation can be mathematically expressed as [12], [15]:

$$\begin{aligned}\varepsilon(p_{fiber}) = (1 - 0.75p_{fiber})I\rho I^\dagger + 0.25p_{fiber}X\rho X^\dagger \\ + 0.25p_{fiber}Y\rho Y^\dagger + 0.25p_{fiber}Z\rho Z^\dagger\end{aligned} \tag{2}$$

Similar to fiber attenuation impacts, error operators due to the quantum nodes are mathematically expressed as [12], [15]:

$$\begin{aligned}\varepsilon(p_{dephase}) = (1 - p_{dephase})I\rho I^\dagger + 0p_{dephase}X\rho X^\dagger \\ + 0p_{dephase}Y\rho Y^\dagger + p_{dephase}Z\rho Z^\dagger\end{aligned} \tag{3}$$

and,

$$\begin{aligned}\varepsilon(p_{depol}) = (1 - 0.75p_{depol})I\rho I^\dagger + 0.25p_{depol}X\rho X^\dagger \\ + 0.25p_{depol}Y\rho Y^\dagger + 0.25p_{depol}Z\rho Z^\dagger\end{aligned} \tag{4}$$

The probability p_{fiber} is a function of length of fiber through which a qubit is transmitted and is expressed as [12], [13], $p_{fiber} = 1 - (1 - p_{init})10^{-\eta l/10}$, here, p_{init} and η being the probability of loss of the qubit immediately after generation and attenuation factor of optical fiber (in dB/km), respectively. Likewise, the qubit decoherence probabilities p_{depol} and $p_{dephase}$ are function of time the qubit spends in a quantum node (Δt) [12], [13] and are given by, $p_{dephase} = 1 - e^{-\Delta t R_{dephase}}$ and $p_{depol} = 1 - e^{-\Delta t R_{depol}}$, where $R_{dephase}$ and R_{depol} denote the dephasing and depolarizing rate, respectively.

B. BiP Calibration Circuit

Quantum error mitigation performs a calibration operation on the outcomes of measurements of the received noisy qubits. The post-processing step of calibration uses a calibration matrix $\mathbf{M}$ that captures the nature of noisy transmission environment (e.g., fiber loss) of the underlying quantum communication system. It relates measurement outcome of a transmitted error-free qubit $\mathcal{C}_{\mathbf{ideal}}$ to that of a received erroneous qubit $\mathcal{C}_{error}$, and the mitigated outcome $\mathcal{C}_{\mathbf{miti}}$ as follows [6]:

$$\mathcal{C}_{\mathbf{error}} = \mathbf{M}\, \mathcal{C}_{\mathbf{ideal}}, \quad \mathcal{C}_{\mathbf{miti}} = \mathbf{M^{-1}}\mathcal{C}_{\mathbf{error}}. \tag{5}$$

Thus, it is important to note that designing the calibration circuit and obtaining the calibration matrix $\mathbf{M}$ are key components of error mitigation methods.

In our proposed BiP error mitigation method, we design the quantum circuit for calibration of the measurement outcomes of the received noisy qubit in three different computational bases as shown in Fig. 2a. The inputs $|+/-\rangle$, $|i/-i\rangle$, and $|0/1\rangle$ are x, y, and z bases states of qubits, respectively. The probabilistic error model (eq. 1) to characterize the environment noise is represented by X, Y, Z, and I Pauli Operators. Additionally, quantum gates H and $S^\dagger$ are used for computing calibration matrices $\mathbf{M}_x$ and $\mathbf{M}_y$ in x and y computational bases. The calibration matrix in z basis, denoted by $\mathbf{M}_z$, is obtained by direct measurement operation after the error model circuit. These calibration matrices are expressed as:

$$\begin{aligned}\mathbf{M}_x &= \begin{pmatrix} p^{|+\rangle}_{(0|x)} & p^{|-\rangle}_{(0|x)} \\ p^{|+\rangle}_{(1|x)} & p^{|-\rangle}_{(1|x)} \end{pmatrix} \\ \mathbf{M}_y &= \begin{pmatrix} p^{|i\rangle}_{(0|y)} & p^{|-i\rangle}_{(0|y)} \\ p^{|i\rangle}_{(1|y)} & p^{|-i\rangle}_{(1|y)} \end{pmatrix} \\ \mathbf{M}_z &= \begin{pmatrix} p^{|0\rangle}_{(0|z)} & p^{|1\rangle}_{(0|z)} \\ p^{|0\rangle}_{(1|z)} & p^{|1\rangle}_{(1|z)} \end{pmatrix}\end{aligned} \tag{6}$$

Each calibration matrix in eq. 6 consists of two columns and two rows. The columns represent the input states $|+/-\rangle$, $|i/-i\rangle$, or $|0/1\rangle$, while the rows correspond to the probability of getting 0 or 1 after the measurement. Thus, matrix element, for instance $p^{|0\rangle}_{(0|z)}$, represents the probability of getting 0 on measuring the received qubit in z computational basis given the input qubit state $|0\rangle$. The comprehensive calibration proposed in the BiP helps in accurately capturing the subtle variations in the angles θ and φ of the received qubit on the Bloch sphere, and thereby, accounting both bit and phase-flips that occurred in this qubit due to noisy environment. The elevation angles θ and φ are used to represent the state of the any qubit $|\psi\rangle(\theta, \varphi)$ on the Bloch Sphere as shown in Fig. 1.

C. Mitigation Process

We infer the measurement outcome of the received qubit in different computational bases using our designed quantum circuit, shown in Fig. 2b. To do so, we use time sequentially three ancillary qubits and obtain the measurement outcomes $\mathcal{C}^x_{error}$, $\mathcal{C}^y_{error}$, and $\mathcal{C}^z_{error}$ in the x, y, z computational bases, respectively. These measurement outcomes are given by:

$$\mathcal{C}^x_{error} = \begin{pmatrix} p^{|\psi^{'}\rangle}_{(0|x)} \\ p^{|\psi^{'}\rangle}_{(1|x)} \end{pmatrix}, \mathcal{C}^y_{error} = \begin{pmatrix} p^{|\psi^{'}\rangle}_{(0|y)} \\ p^{|\psi^{'}\rangle}_{(1|y)} \end{pmatrix}, \mathcal{C}^z_{error} = \begin{pmatrix} p^{|\psi^{'}\rangle}_{(0|z)} \\ p^{|\psi^{'}\rangle}_{(1|z)} \end{pmatrix} \tag{7}$$

with $|\psi^{'}\rangle$ being the received qubit state with errors shown in Fig.1. This received qubit can be expressed in terms of angles with respect to different axis of the Bloch sphere as follows:

$$|\psi^{'}\rangle = cos(\theta'/2)|0\rangle + e^{i\varphi'} sin(\theta'/2)|1\rangle \tag{8}$$

Combining equations 5, 6, and 7, we get the mitigated measurement outcomes and state $|\psi_{miti}\rangle$ as:

$$\mathcal{C}^x_{miti} = \begin{pmatrix} p^{|\psi_{miti}\rangle}_{(0|x)} \\ p^{|\psi_{miti}\rangle}_{(1|x)} \end{pmatrix}, \mathcal{C}^y_{miti} = \begin{pmatrix} p^{|\psi_{miti}\rangle}_{(0|y)} \\ p^{|\psi_{miti}\rangle}_{(1|y)} \end{pmatrix}, \mathcal{C}^z_{miti} = \begin{pmatrix} p^{|\psi_{miti}\rangle}_{(0|z)} \\ p^{|\psi_{miti}\rangle}_{(1|z)} \end{pmatrix} \tag{9}$$

$$|\psi_{miti}\rangle = cos(\theta_{miti}/2)|0\rangle + e^{\varphi}_{miti} sin(\theta_{miti}/2) \tag{10}$$

For the qubits $|\psi'\rangle$, $|\psi_{miti}\rangle$, the logical relation between the measurement outcomes and the elevation angles are summarized as:

$$\begin{pmatrix} cos^2(\theta'/2) \\ (e^{i\varphi'} sin(\theta'/2))^2 \end{pmatrix} = \begin{pmatrix} p^{|\psi'\rangle}_{(0|z)} \\ p^{|\psi'\rangle}_{(1|z)} \end{pmatrix} \tag{11}$$

$$\begin{pmatrix} cos^2(\theta_{miti}/2) \\ (e^{i\varphi_{miti}} sin(\theta_{miti}/2))^2 \end{pmatrix} = \begin{pmatrix} p^{|\psi_{miti}\rangle}_{(0|z)} \\ p^{|\psi_{miti}\rangle}_{(1|z)} \end{pmatrix} \tag{12}$$

The elevation angle of qubit along z coordination in Bloch sphere, $\theta^{'}$ or θ_{miti}, has a range between $0°$ and $180°$.

However, these two equations 11 and 12 are insufficient for computing the phase information of qubits' states, represented by $\varphi^{'}$ or φ_{miti}, which are azimuth angle along x coordination of the Bloch sphere. To get this phase information, we rewrite $e^{i\varphi^{'}}$ and $e^{i\varphi_{miti}}$ as trigonometric expressions with Euler equation. Considering the relation between elevation and azimuthal angle in a Spherical coordinate system, we get:

$$\begin{pmatrix} cos^2(acos(cos(\varphi^{'})sin(\theta^{'})/2) \\ sin^2(acos(cos(\varphi^{'})sin(\theta^{'})/2) \end{pmatrix} = \begin{pmatrix} p^{|\psi^{'}\rangle}_{(0|x)} \\ p^{|\psi^{'}\rangle}_{(1|x)} \end{pmatrix} \tag{13}$$

$$\begin{pmatrix} cos^2(acos(cos(\varphi_{miti})sin(\theta_{miti})/2) \\ sin^2(acos(cos(\varphi_{miti})sin(\theta_{miti})/2) \end{pmatrix} = \begin{pmatrix} p^{|\psi_{miti}\rangle}_{(0|x)} \\ p^{|\psi_{miti}\rangle}_{(1|x)} \end{pmatrix} \tag{14}$$

Here, $\varphi^{'}$ or φ_{miti} has range of $-180°$ to $180°$. Thus, from equations 13 and 14 we get magnitude $|\varphi^{'}|$ or $|\varphi_{miti}|$.:

We utilize the angle β ($0°$ to $180°$) along y coordinate of the Bloch sphere to confirm the sign of $|\varphi^{'}|$ or $|\varphi_{miti}|$ as follows:

$$\begin{pmatrix} cos^2(\beta^{'}/2) \\ sin^2(\beta^{'}/2) \end{pmatrix} = \begin{pmatrix} p^{|\psi^{'}\rangle}_{(0|y)} \\ p^{|\psi^{'}\rangle}_{(1|y)} \end{pmatrix} \tag{15}$$

$$\begin{pmatrix} cos^2(\beta_{miti}/2) \\ sin^2(\beta_{miti}/2) \end{pmatrix} = \begin{pmatrix} p^{|\psi_{miti}\rangle}_{(0|y)} \\ p^{|\psi_{miti}\rangle}_{(1|y)} \end{pmatrix} \tag{16}$$

As a summary, the state of the received qubit with error and mitigation through equations 7 to 16 can be expressed as:

$$\varphi^{'} = \begin{cases} acos(cos(2 \times acos\left(\sqrt{p^{|\psi^{'}\rangle}_{(0|x)}}\right))/sin(\theta^{'})), \\ \text{if}\beta^{'} = [0°, 90°] \\ -acos(cos(2 \times acos\left(\sqrt{p^{|\psi^{'}\rangle}_{(0|x)}}\right))/sin(\theta^{'})), \\ \beta^{'} = [90°, 180°] \end{cases} \tag{17}$$

$$\varphi_{miti} = \begin{cases} acos(cos(2 \times acos\left(\sqrt{p^{|\psi_{miti}\rangle}_{(0|x)}}\right))/sin(\theta_{miti}), \\ \text{if } \beta_{miti} = [0°, 90°] \\ -acos(cos(2 \times acos\left(\sqrt{p^{|\psi_{miti}\rangle}_{(0|x)}}\right))/sin(\theta_{miti}), \\ \text{if } \beta_{miti} = [90°, 180°] \end{cases} \tag{18}$$

IV. Results and Discussions

In this section, we evaluate our proposed BiP error mitigation method using the theoretical model described in Section III. We use QISKIT library (version 0.38.0) with Python version 3.8 for running simulations on an Intel Core i7-9750H CPU processor. We evaluate the performance of our proposed BiP-based error mitigation method for any random qubit received from a noisy transmission channel in terms of mitigation on the measurement outcome, elevation and azimuthal angle, fidelity of the received qubit, and the processing time. As shown in error model of Section III-A, equations 1-**??** demonstrate the connection between environment and the error model. Based on these equations, the error models are built at the QISKIT platform. Typically, we set the parameter values as: $p_{init} = 0.009$, fibre attenuation $\eta = 0.15$ dB/km, dephasing and depolarization rate $R_{dephase} = 10^3, R_{dephase} = 10^6$, and $\Delta t = 10$ ns [12]. Given the transmitted qubit $|\psi\rangle(\theta = 120°, \varphi = 60°)$, we first consider an optical fiber with 100 km to validate the efficacy of BiP on the measurement and rotation angles. Finally, the effect of the mitigation approach on the fidelity are shown and compared for different fiber lengths.

During error mitigation, the quantum circuits used in the simulation for the purpose of producing the calibration matrix and the measurement of received qubit with and without noise are depicted in Fig.2. These are implemented within the *Aer* simulator engine in the QISKIT platform at different sampling time. The sampling time represents the number of times the quantum circuit is invoked and this can be specified via the 'shots' argument in the *Aer* simulator. The shots argument is directly related the accuracy stability of the measurement during different simulation rounds. Specifically, higher value of the shots argument in the simulator ensures convergence to identical values for different rounds. While this ensures high accuracy it incurs the cost of longer execution time. In our work, we run our simulation in a range of times of corresponding to 1000 - 35000 shots. The averaged results for different shots are represented in the following figures.

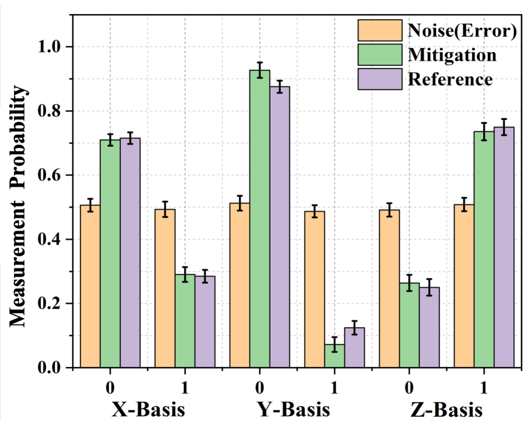


Fig. 3: Mitigation on the measurement outcomes of the received qubit.

A. Performance of Mitigation on Measurement Outcomes

We initiate the BiP error mitigation by measuring the received qubit in the x, y, and z computational basis and averaging the measurement outcomes obtained in different shots to compute the probability of getting $0/1$ after measurement. We compute these probabilities for three cases: 1) error-free qubit considered as *reference*, 2) erroneous qubit received from the environment representing *noise (error)*, and 3) qubit after applying BiP-based *mitigation*, as shown in Fig. 3. We observe that the difference in probabilities of measuring $0/1$ for the mitigation and reference case is less than 5%. This validates the efficacy of our proposed BiP error mitigation method in mitigating the environment noise from the received qubit.

B. Mitigation of Rotation in Angles

In Fig. 4, we show the rotation in angles of the received qubits for the three cases mentioned in Sec.III-B. These rotations are calculated from the probability of measuring 0/1 for the received qubit. We observe that after mitigating rotation in angles θ and φ using BiP, their respective values are close to the ones obtained in the transmitted qubit (reference). This highlights that the BiP is able to accurately mitigate changes in angles of the noisy received qubit.

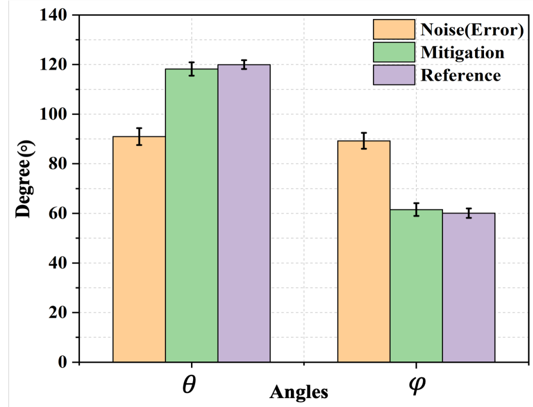


Fig. 4: Mitigation of the elevation and azimuthal angles of the received qubit.

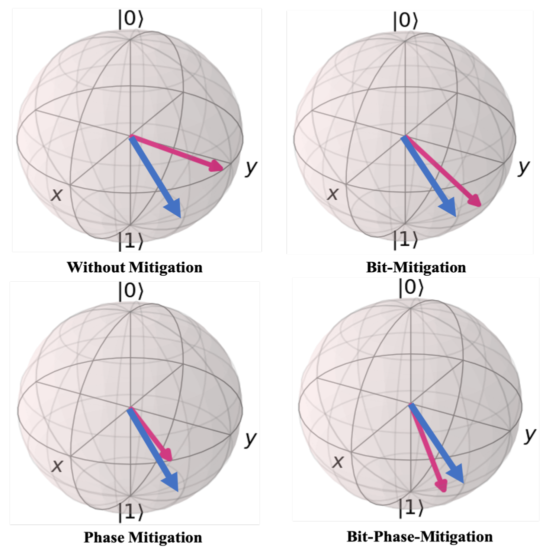


Fig. 5: Visual representation of the received qubit after mitigation of errors using different methods: no mitigation, phase-flip mitigation, bit-flip mitigation, and bit-phase-flip mitigation.

C. Comparison of Received Qubit's Fidelity Achieved Using Different Mitigation Methods

The fidelity of a qubit is a commonly used metric for evaluating its quality. It represents similarity between the received qubit and the transmitted qubit [13]. In this section, we give the comparison of our proposed BiP-based error mitigation with other mitigation methods in terms of improving the fidelity of the qubit received from the noisy environment. We consider no mitigation, bit-based and phase-based mitigation methods for this comparison. In Fig. 5, we give a visual representation of the mitigated qubit on a Bloch sphere for the considered methods. The red arrows represent the transmitted qubit states, and the blue arrows indicate the mitigated qubit states. The fidelity in Bloch Sphere can be understood as the projection of the received qubit on transmitted one. Thus, the smaller angle between the transmitted and received qubit states implies better fidelity of the received qubit and thus, better error mitigation. It can be observed that this angle is minimum in case of BiP.

Further, we quantify the fidelity of the received qubits using different methods in Fig. 6, which illustrates significant improvement in quality of the received qubit using BiP. We perform this experiment for different noise models by changing the length of transmission channels (50 km and 100 km). We notice that BiP is more robust to noise as it provides comparable improvement in fidelity of the qubits received over different channel lengths, as opposed to other methods.

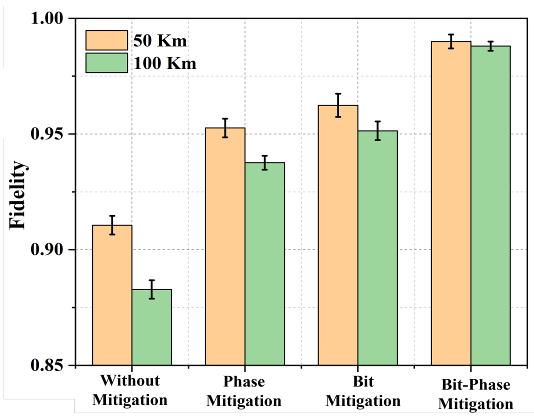


Fig. 6: Performance comparison of different error mitigation methods in providing improvement in the fidelity of the received qubit for different transmission channel lengths.

D. Comparison of Processing Time

In Fig.7, we provide the processing time required for completing the calibration and the measurement steps for different shots argument in the Aer simulator. We observe that changing the length of fiber has no impact on the processing time and thus, we use a fiber of length 100 km for this simulation. The figure shows that the processing time increases linearly with the number of shots argument in the simulation, which is intuitive. An interesting future direction is to reduce this processing time by designing less-complex quantum circuits.

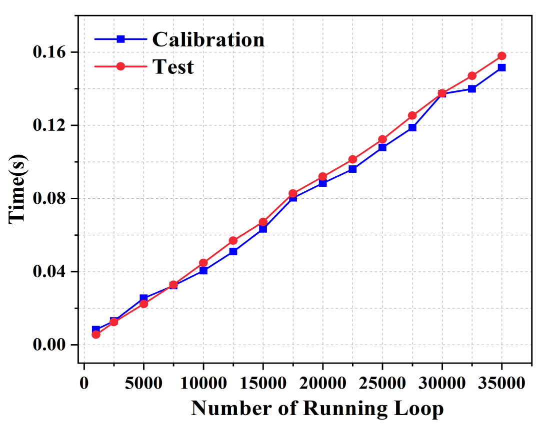


Fig. 7: Processing time with different number of shots of run during the simulation.

V. Conclusion

In this paper, we propose an error mitigation method, called BiP, by fully characterizing the bit and phase flip errors that arise due to noise present in a quantum communication system. These errors are modeled using Pauli operators. Our approach computes different calibration matrices and performs measurements of the received qubit in different x, y, and z computational bases to acquire its bit and phase information. Importantly, it does not require any additional hardware and complex encoding protocol for correcting errors in qubits, which is common approach used in the quantum error correction. The results demonstrate that the BiP accurately mitigates the errors by correcting the elevation and azimuthal angles, and improving the fidelity of the qubit received from noisy transmission channel. Thus, our proposed BiP-based error mitigation has potential to address the environment noise inherent in large-scale quantum communication systems.

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